Chapter 6

BOND VALUATION

A Bond/debenture is a debt instrument issued by a government/company. It is issued by the issuer for raising long term loan funds. From the investor point of view, it is a fixed income provider, relatively less-risky, investment.

**Coupon rate:** It is the rate of interest that is given by the issuer to the bondholder. The interest is calculated on face value. For example if the coupon rate is 10%, it means interest is paid annually at the rate of 10% on face value. If it is 12% semiannual, it means interest is paid half-yearly at the rate of 6% on the face value.

<table>
<thead>
<tr>
<th>YIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current yield:</strong>&lt;br&gt;(Annual interest based on coupon rate / current market price of the Bond) x 100</td>
</tr>
</tbody>
</table>

---

1 In case the instrument is partly paid, interest is calculated on the paid up value.
Q. No. 1: Face value of a bond Rs.1000, coupon rate 6%, Current market price Rs.900. Current Yield?

Answer
Current Yield = \( \frac{60}{900} \times 100 = 6.67\% \)

Yield to maturity\(^2\): It is the annualized rate of return on the investment that the investor expects (on the date of investment) to earn from the date of the investment to the date of maturity. It is also referred as the required rate of return\(^3\). Theoretically, it is equal to current market interest rate\(^4\).

YTM - Regular interest bonds (Non-zero coupon bonds)

Q. No. 2: A 4 years debenture with 10% coupon rate, maturity value Rs.1,000, in currently selling at Rs.900. YTM?

Answer
Average return per year per debenture: \( \frac{(400+100)}{4} \) = 125
Approximate annual rate = \( \frac{125}{900} \times 100 = 13.89\% \)

NPV at 14% = \(-900 + 100(2.914) + 1000(0.592)\) = -16.60
As NPV (at 14%) is negative, this shows that the return is less than 14%.

Let calculate NPV at 13%.

NPV at 13% = \(-900 + 100(2.974) + 1000(0.613)\) = +10.40
As NPV (at 13%) is positive, this shows that the return is greater than 13%. We can find the exact return (YTM, also called as current interest rate) through interpolation.

YTM or current interest rate:

\[
\text{YTM} = \frac{\text{Lower rate} \times \text{NPV}}{\text{NPV} - \text{Higher rate} \times \text{NPV}}
\]

\[
10.40 = 13 + \frac{10.40}{-16.60} \times 1 = 13.386\%
\]

\(^2\) Unless the question requires otherwise, it should be calculated on pre-tax basis i.e. the tax should be ignored for this calculation.

\(^3\) Unless the question requires otherwise, it should be calculated on pre-tax basis i.e. the tax should be ignored for this calculation.

\(^4\) It is always pre-tax.
Q. No. 3 : A company has outstanding 8 per cent debentures of Rs.10,00,000 on which interest is payable annually on 31 December. The debentures are due for redemption at par on 1.1.1993. The market price of debenture at 31.12.1989 was Rs.103 cum-interest. Ignore Tax. What do you estimated to be current market rate of interest? (This is also called yield to Maturity.)

Answer
Average return per year per debenture: \([\frac{(24+5)}{(3)}]\) = \(9.67\)
Approximate annual rate = \([\frac{9.67}{95}]\times100 = 10.18\%\)

NPV at 10 % = \(-95 + (8 \times 0.909) + (8 \times 0.826) + (108 \times 0.751) = -0.012\)
As NPV (at 10%) is negative, this shows that the return is less than 10%. Let calculate NPV at 9%.

NPV at 9 % = \(-95 + (8 \times 0.917) + (8 \times 0.842) + (108 \times 0.772) = + 2.448\)
As NPV (at 9%) is positive, this shows that the return is greater than 9%. We can find the exact return (YTM, also called as current interest rate) through interpolation.

\[
\text{YTM or current interest rate} = \frac{\text{Lower rate} \times \text{NPV}}{\text{NPV} - \text{Higher rate NPV}}
\]

\[
2.448 = 9 + \frac{\text{NPV}}{2.448 - (-0.012)} = 9.995\%
\]

Q. No. 4 : There is a 9% 5-year bond issue in the market. The issue price is Rs.90 and the redemption price is Rs.105. For an investor with marginal tax rate of 30% and capital gain tax rate of 10% (assuming no indexation), what is the post tax yield to maturity? (May, 2004)

Answer
Let’s assume that the face value of the bond is Rs.100.
Average return per year per Bond: \([(31.50 + 13.50) / (5)] = 9\)
Approximate post tax annual yield = \([\frac{9.00}{90}]\times100 = 10\%\)

NPV at 10 % = \(-90 + (6.3 \times 3.791) + (103.50 \times 0.621) = -1.8432\)
As NPV (at 10%) is negative, this shows that the return is less than 10%.

Let calculate NPV at 9%.

NPV at 9 % = \(-90 + (6.3 \times 3.89) + (103.50 \times 0.650) = +1.782\)
As NPV (at 9%) is positive, this shows that the return is greater than 9%. We can find the exact return (called YTM, also called current interest rate) through interpolation.

YTM or current interest rate:

\[
\text{YTM or current interest rate:}
\]

\[
\begin{align*}
\text{Lower rate NPV} &= \text{Lower rate} + \frac{\text{NPV}}{\text{Higher rate NPV}} \times (\text{difference in rates}) \\
&= 9 + \frac{1.782}{1.782 - (-1.8432)} \times 1 = 9.49%
\end{align*}
\]

YTM – Zero Coupon Bonds

**Q. No. 5:** A company issues Zero coupon bond of 10 years maturity. Issue price Rs.260. Maturity value Rs.1000. Ignore tax. YTM?

**Answer**

Present value of Rs.1,000 to be received after 10 years = Rs.260

PV of Re.1 to be received after 10 years = 0.26. Consulting the PVF table, we find that the rate of interest in this case is in the range of 14% to 15%.

NPV at 14 % = -260 + (1000 x 0.270) = + 10

As NPV (at 14%) is positive, this shows that the return is greater than 14%.

Let calculate NPV at 15%.

NPV at 15 % = -260 + (1000 x 0.247) = -13

As NPV (at 15%) is negative, this shows that the return is less than 15%. We can find the exact return (called YTM, also called current interest rate) through interpolation.

YTM or current interest rate:

\[
\begin{align*}
\text{Lower rate NPV} &= \text{Lower rate} + \frac{\text{NPV}}{\text{Higher rate NPV}} \times (\text{difference in rates}) \\
&= 14 + \frac{10}{10 - (-13)} \times 1 = 14.43%
\end{align*}
\]

**Q. No. 6:** What is the YTM of Rs.1000 10 years zero coupon bond if the issue price Rs.190?

**Answer**
• Present value of Rs.1,000 to be received after 10 years = Rs.190
• PV of Re.1 to be received after 10 years = 0.19.
• Consulting the PVF table, we find that the rate of interest in this case is in the range of 18% to 19%.
• NPV at 18% = -190 + (1000 x 0.191) = + 1
• As NPV (at 18%) is positive, this shows that the return is greater than 18%.

Let calculate NPV at 19%.

• NPV at 19% = -190 + (1000 x 0.176) = - 14
• As NPV (at 19%) is negative, this shows that the return is less than 19%.

We can find the exact return (called YTM, also called current interest rate) through interpolation.

**YTM or current interest rate:**

\[
\text{YTM or current interest rate:} = \frac{1}{1 - (-14)} = 18.066% \\
\text{NPV at 14% = -950 + (100 x 2.322) + (1050 x 0.675) = - 9.05} \\
\text{As NPV (at 14%) is negative, this shows that the return is less than 14%.

Yield to Call : Sometimes, the terms of issue of bonds contain a provision for call option, i.e. issuer has the option of calling (buying) the bonds for redemption before the date of maturity of the bonds. Yield to Call refers to the annualized rate of return on the investment that the investor expects (on the date of investment) to earn from the date of the investment to the date of call.

Q. No. 7 : A Bond is currently traded at Rs.950. Its face value is Rs.1000. Coupon rate is 10%. It is redeemable at par after 5 years from today. However the company has an option of calling it after 3 years from today at 5% premium. Find Yield To Call.

**Answer**

Average return per year per Bond (till call): [(400)/(3)] = 133.33
Approximate annual yield = [133.33 / 950]x100 = 14.04 %

• NPV at 14% = -950 + (100 x 2.322) + (1050 x 0.675) = - 9.05
• As NPV (at 14%) is negative, this shows that the return is less than 14%.

Let calculate NPV at 13%.

• NPV at 13% = -950 + (100 x 2.361) + (1050 x 0.693) = + 13.75
• As NPV (at 13%) is positive, this shows that the return is greater than 13%.
We can find the exact return (called YTM, also called current interest rate) through interpolation.

YTM or current interest rate:

\[
\text{Lower rate} \quad \text{NPV} \\
= \text{Lower rate} + \frac{\text{difference in rates}}{(\text{Lower rate} \quad \text{NPV} - \text{Higher rate} \quad \text{NPV})} \times 1
\]

\[
= 13 + \frac{13.60\%}{13.75 - (\text{-9.05})} = 13.60\%
\]

**VALUE OF BOND**

It is the present value of all future cash inflows arising on account of the bond to the investor. The present value is being calculated using the bond’s yield-to-maturity as the discount rate. It is also referred as required rate/expected rate/yield/return to maturity/opportunity cost. Theoretically, it is equal to current market interest rate\(^5\).

**Q. No. 8 :** A Ltd. has in issue two debentures. It is strong company with a good profit record. The debentures are:

(i) 6% irredeemable debentures (face value Rs.100).

(ii) 7% debentures redeemable in four years’ time at par (face value Rs.100).

The current market rate of interest on investment of this kind is 9 per cent. Find the market value of each of these two debentures.

**Answer (i)**

Market value of debenture = \(\frac{6}{1.09} + \frac{6}{(1.09)^2} + \frac{6}{(1.09)^3} + \cdots\infty\)

\[
= \frac{6}{1- (1/1.09)} = 5.5046 = \frac{66.64}{0.0826} = \text{Rs.66.64}
\]

**Answer (ii)**

Market value of debenture = \(\frac{7}{1.09} + \frac{7}{(1.09)^2} + \frac{7}{(1.09)^3} + \frac{107}{(1.09)^4} = 93.52\)

**Q. No. 9 :** M/s Agfa is planning to issue a debentures series on the following terms:

<table>
<thead>
<tr>
<th>Years</th>
<th>Yearly coupon rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>9%</td>
</tr>
<tr>
<td>5-8</td>
<td>10%</td>
</tr>
<tr>
<td>9-10</td>
<td>14%</td>
</tr>
</tbody>
</table>

\(^5\) It is always pre-tax.
The current market rate on similar debentures is 15% p.a. The company proposes to price the issue in such a manner that it can yield 16% compounded rate of return to the investors. The company also proposes to redeem the debentures at 5% premium on maturity. Determine the issue price of the debentures. (Nov. 2003)

**Answer**

Issue price = $9(2.798) + 10(1.546) + 14(0.263) + 119(0.227) = 71.34$

Q. No. 10 : RBI sold a 91 day T -bill of face value of Rs.100 at an yield of 6% p.a. Issue price? (May, 2005)

**Answer**

Interest rate for 91 days = 6 x (91/365) = 1.4958904
Issue price = $100 / 1.014958904 = Rs.98.53$

Q. No. 11 : A money market instrument with face value of Rs.100 and discount yield of 6% will mature in 45 days. You are required to calculate:

(i) Current price of the instrument
(ii) Bond Equivalent yield
(iii) Effective annual return. (May, 2007)

**Answer**

Let’s assume that 1 year = 360 days

(i) Interest of Re 1 for 45 days = 0.06 x 45/360 = 0.0075
Let current price = $X$
$X (1.0075) = 100$
$X = 99.26$

(ii) Bond Equivalent yield is calculated as per annum.
Hence Bond yield = $(0.0075) x (360/45) = 6$

(iii) Effective Yield is calculated as compounded interest i.e. interest compounding every period of interest.
Hence Effective Interest = $(1.0075)^{360/45} - 1$
$= (1.0075)^{360/45} - 1 = .0616 = 6.16$

Q. No. 12 : The 6-months forward price of a security is Rs.208.18. The interest is 8% p.a. with monthly rests. What is the spot price? (Nov. 2006)

**Answer**
Let spot price = x
\[ x(1.0066667)^6 = 208.18 \]
x = Rs.200.04

Q. No. 13: MP Ltd issued a new series of bonds on 1st January, 2000. The bonds were sold at par (Rs.1000), having a coupon rate of 10% p.a. and mature on 31st December, 2015. Coupon payments are made semiannually on 30th June and 31st Dec each year. Assume that you purchased an outstanding bond on 1st March, 2008 when the on going interest rate was 12%. Required:

(i) What was the YTM of the bonds as on 1st January, 2000?

(ii) What amount you should pay to complete the transaction? Of that amount how much should be accrued interest and how much would represent bond’s basic value.

(Nov. 2007)

Answer
(i) YTM on 1st January 2000 = 10%
(ii) Assumption: Current interest rate on 1st January 2008 = 12% p.a.
    semiannually.
    Market value on 1st January 2008 :
    \[ 50(9.712) + 1050(0.394) = 899.30 \]
    Market value on 1st March, 2008 : 899.30(1.02) = 917.29
    Payment for complete transaction : 917.29
    Interest accrued = 1000 x 0.10 x (2/12) = 16.67
    Bond’s basic value = 917.29 – 16.67 = 900.62

Bond Risk
The term risk is used to denote the possibility of variability in the returns expected from the investment i.e. the actual return differs from the expected one. Investment in bonds is not entirely risk free. Both systematic and unsystematic risks are associated with the investment in bonds. Unsystematic risk, in case of bonds, refers to default risk, i.e. the issuer may default in the payment of interest or principal or both on the stipulated dates. Systematic risk arising on the investment on bonds is referred as interest risk. Interest risk refers to change in market interest rate during the holding period. Change in the interest rate has two effects (i) the investor shall be reinvesting the interest income of the bond at the changed rate and (ii) Market price of the bond will change. If the market interest rate moves up, on one hand the investor shall be benefited as he shall be able to reinvest the interest income at the higher rate and on the other hand he will suffer loss as the market value of the investment will go down. (Remember that the bond prices move inversely to market interest rate changes.) If the market interest rate goes down, on one hand he will suffer loss as he shall be able to reinvest the interest income at lower rate and the other hand he will be benefited by upward movement in the market price of the investment.

| DURATION |
Duration is the weighted average time to receive the present value of bond. The weights are the present values of the payments, using the bond’s yield-to-maturity as the discount rate. (Duration is also called as Macaulay Duration)

Let’s understand the concept of duration with the help of an example:

**Example:** Suppose a person lends Rs.3,20,000 to his friend on interest free basis. The friend returns him Rs.10,000 at the end of 1\textsuperscript{st} year, Rs.10,000 at the end of 2\textsuperscript{nd} year and Rs.3,00,000 at the end of 3\textsuperscript{rd} year. What is average period of the loan?

**Answer**

<table>
<thead>
<tr>
<th>X</th>
<th>W</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>2</td>
<td>10,000</td>
<td>20,000</td>
</tr>
<tr>
<td>3</td>
<td>3,00,000</td>
<td>9,00,000</td>
</tr>
</tbody>
</table>

\[\sum W = 320000 \quad \sum XW = 9,30,000\]

Weighted average period = \(\frac{\sum XW}{\sum W} = \frac{9,30,000}{320000} = 2.90625\)

In case interest is considered, weights being present value of cash flows arising from the investment.

**Q. No. 14:** The following data are available for a bond: Face value Rs.1000. Coupon rate 6%. Years to maturity 6. Redemption at par YTM = 17%. Find the current price, and duration of the bond.

**Answer:**

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 x 0.855</td>
<td>51.30</td>
</tr>
<tr>
<td>2</td>
<td>60 x 0.731</td>
<td>87.72</td>
</tr>
<tr>
<td>3</td>
<td>60 x 0.624</td>
<td>112.32</td>
</tr>
<tr>
<td>4</td>
<td>60 x 0.534</td>
<td>128.16</td>
</tr>
<tr>
<td>5</td>
<td>60 x 0.456</td>
<td>136.80</td>
</tr>
<tr>
<td>6</td>
<td>1060 x 0.390</td>
<td>2480.40</td>
</tr>
</tbody>
</table>

\[\sum W = 605.40 \quad \sum XW = 2996.70\]

Current price = PV of all cash in flows = 605.40
Duration = \(\frac{\sum XW}{\sum W} = \frac{2996.70}{605.40} = 4.95\)

Duration of a non-zero coupon bond is always shorter than its maturity. Duration of a zero-coupon bond is exactly equal to its maturity.

**Q. No. 15:** A 3 years maturity zero coupon bond is currently sold at Rs.816. Its maturity value is Rs.1000. Find its duration.

**Answer**

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
</table>
Volatility

Volatility refers to the sensitivity of the bond price to change in current interest rate. Duration is used to measure the sensitivity of the bond price to changes in interest rates.

Volatility is measured with the help of Modified duration. Modified duration is equal to “[Duration/(1+YTM/n)]”, where n is number of interest payments in a year. Suppose, Modified duration is 3, it means if the Current interest rate changes by 100 basis points, the price of bond will change by 3% in opposite direction.

% change in bond price = - [Duration / (1+ YTM/n)] x (Δ BP/100)

Q. No. 16 : Find the volatility of the bond using the data of Q. No.15. If YTM increases by 100 basis points, what will be new current price of the bond?

Answer
Volatility : -(3/1.07) = - 2.8037
New current price = 816 x 0.971963 = Rs.793.12

Q. No. 17 : Current price of bond Rs.950. Current rate 10 %. Duration of the bond is 3 years. If interest rate changes to 11 %, what will be the price of the bond?

Answer
% change in bond price = - [3/(1+.10)] x (+ 1) = -2.73.
New bond price = 950x.9727 i.e. Rs.924.065

Q. No. 18 : Determine the duration of a bond which has face value of Rs. 1000, coupon rate 8% annual, maturity 4 years, YTM 10%. What is the modified duration? If the YTM goes up from 10 % to 11%, determine the new price of the bond.

Answer

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80 x 0.909</td>
<td>72.72</td>
</tr>
<tr>
<td>2</td>
<td>80 x 0.826</td>
<td>132.16</td>
</tr>
<tr>
<td>3</td>
<td>80 x 0.751</td>
<td>180.24</td>
</tr>
<tr>
<td>4</td>
<td>1080 x 0.683</td>
<td>2950.56</td>
</tr>
<tr>
<td>SUM</td>
<td>936.52</td>
<td>SUM W=936.52</td>
</tr>
<tr>
<td>SUM</td>
<td>3335.68</td>
<td>SUM XW=3335.68</td>
</tr>
</tbody>
</table>

Duration = SUM XW / SUM W = 3335.68/936.52 = 3.56
Modified duration = -[3.56/1.10] = - 3.24 New price = 936.52 x 0.9676 = 906.18
Q. No. 19 : Find the current market price of a bond having face value of Rs.1,00,000 redeemable after 6 year maturity with YTM at 16% payable annually and duration 4.3203 years. Given $1.16^6 = 2.4364$. (May, 2007)

**Answer**

Let annual interest = c

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>c x 0.862</td>
<td>0.862c</td>
</tr>
<tr>
<td>2</td>
<td>c x 0.743</td>
<td>1.486c</td>
</tr>
<tr>
<td>3</td>
<td>c x 0.641</td>
<td>1.923c</td>
</tr>
<tr>
<td>4</td>
<td>c x 0.552</td>
<td>2.208c</td>
</tr>
<tr>
<td>5</td>
<td>c x 0.476</td>
<td>2.380c</td>
</tr>
<tr>
<td>6</td>
<td>c x 0.410</td>
<td>2.460c</td>
</tr>
<tr>
<td></td>
<td>1,00,000x 0.410</td>
<td>2.46,000</td>
</tr>
<tr>
<td></td>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>6</td>
<td>3.684c + 41000</td>
<td>246000 + 11.319c</td>
</tr>
</tbody>
</table>

\[
4.3203 = \frac{(246000 + 11.319c)}{(3.684c + 41000)}
\]

\[
15.915985c + 177132.3 = 246000 + 11.319c
\]

\[
4.596985c = 68867.7
\]

C = 14981 say 15000  
Coupon rate = 15%

Current price of debenture = 15000 x 3.685 + 41000 = 96275

Q. No. 20: Duration of a bond is 4.50 years. YTM = 8 % p.a. payable half yearly. Find the % change in its price if the YTM declines from 8 % to 7%.

**Answer**

\[
\% \text{ change in bond price} = - \left[ \frac{4.50}{1.04} \right] \times (-1) = + 4.33\%
\]

Q. No. 21: The investment portfolio of a bank is as follows:

<table>
<thead>
<tr>
<th>Government Bond</th>
<th>Coupon rate</th>
<th>Purchase rate</th>
<th>Duration (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOI 2009</td>
<td>11.68</td>
<td>106.50</td>
<td>3.50</td>
</tr>
<tr>
<td>GOI 2013</td>
<td>7.55</td>
<td>105.00</td>
<td>6.50</td>
</tr>
<tr>
<td>GOI 2018</td>
<td>7.38</td>
<td>105.00</td>
<td>7.50</td>
</tr>
<tr>
<td>GOI 2025</td>
<td>8.35</td>
<td>110.00</td>
<td>8.75</td>
</tr>
<tr>
<td>GOI 2035</td>
<td>7.95</td>
<td>101.00</td>
<td>13.00</td>
</tr>
</tbody>
</table>

Face value of total investment is Rs.5 Crores in each Government bond. Calculate actual investment in portfolio.
What is a suitable action to churn out investment portfolio in the following scenario?

1. interest rates are expected to lower by 25 basis points
2. interest rates are expected to rise by 75 basis points

Also calculate the revised duration of investment portfolio in each scenario.

(Oct. 2005)

**Answer**

Calculation of investment in each security

<table>
<thead>
<tr>
<th>Security</th>
<th>Purchase price</th>
<th>Investment (Rs. Crores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOI 2009</td>
<td>Rs.106.50</td>
<td>5.3250</td>
</tr>
<tr>
<td>GOI 2013</td>
<td>Rs.105.00</td>
<td>5.2500</td>
</tr>
<tr>
<td>GOI 2018</td>
<td>Rs.105.00</td>
<td>5.2500</td>
</tr>
<tr>
<td>GOI 2025</td>
<td>Rs.110.00</td>
<td>5.5000</td>
</tr>
<tr>
<td>GOI 2035</td>
<td>Rs.101.00</td>
<td>5.0500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26.3750</td>
</tr>
</tbody>
</table>

Teaching note – not to be given in the exam. We should understand the following point before answering this question:

If interest rate (current interest rate in the market) changes, market price of the bond changes in the reverse direction. Larger the duration, larger the change (the opposite direction), smaller the duration, smaller the change.

In case the interest rate is expected to lower, the price of the bonds will rise.
- There will be larger increase in case of bonds with larger duration. Hence such bonds should be purchased. For example GOI 2035 Bonds may be purchased. (even GOI 2025 may be purchased)
- There will be lower increase in case of bonds with lower duration. Hence such bonds may be sold. For example GOI 2009 Bonds may be sold. (even GOI 2013 may be sold)

In case the interest rate is expected to rise, the price of the bonds will lower.
- There will be larger decrease in case of bonds with larger duration. Hence such bonds should be sold. For example GOI 2035 Bonds may be sold. (even GOI 2025 may be sold)
- There will be smaller decrease in case of bonds with lower duration. Hence such bonds may be purchased. For example GOI 2009 Bonds may be purchased. (even GOI 2013 may be purchased)

Q. No. 22: John inherited the following securities on his uncle’s death:

<table>
<thead>
<tr>
<th>Type of security</th>
<th>Nos.</th>
<th>Annual coupon %</th>
<th>Maturity years</th>
<th>Yield %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond A</td>
<td>10</td>
<td>9</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>
(Rs.1,000)  
<table>
<thead>
<tr>
<th>Bond B</th>
<th>10</th>
<th>10</th>
<th>5</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Rs.1,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preference shares C</td>
<td>100</td>
<td>11</td>
<td>*</td>
<td>13*</td>
</tr>
<tr>
<td>(Rs.100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preference shares D</td>
<td>100</td>
<td>12</td>
<td>*</td>
<td>13*</td>
</tr>
<tr>
<td>(Rs.100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Likelihood of being called at a premium over par.
Compute the current value of the portfolio.

**Answer**

In the absence of information regarding early call, this point has been ignored.

Preference shares have been assumed to be irredeemable.

Value of portfolio:

<table>
<thead>
<tr>
<th>Security</th>
<th>Value per security</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond A</td>
<td>[90/(1.12)] + [90/(1.12)^2] + [1090/(1.12)^3] = 928</td>
<td>9,280</td>
</tr>
<tr>
<td>Bond B</td>
<td>[100/(1.12)] + [100/(1.12)^2] + ... + [1100/(1.12)^5] = 928</td>
<td>9,280</td>
</tr>
<tr>
<td>PS C</td>
<td>[11/(1.13)] + [11/(1.13)^2] + ... = 84.60</td>
<td>8,460</td>
</tr>
<tr>
<td>PS D</td>
<td>[12/(1.13)] + [12/(1.13)^2] + ... = 92.30</td>
<td>9,230</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>36,250</td>
</tr>
</tbody>
</table>

**YIELD CURVE**

An yield curve is a graphic representation of term structure of returns on bonds.

It is a graphic representation of yields (at a particular point of time) on similar quality fixed income instruments plotted against various maturities i.e. the curve exhibits yields currently available on similar type of bonds of different maturities. For drawing the yield curve, yields of comparable securities, at a particular point of time, are plotted on Y-axis and their maturities (time till redemption) are plotted on X-axis.

With the help of yield curve, an investor can easily find the yields that are available on the comparable fixed income securities for short-period, medium-period and long-period. For example, the securities of Government of India are the fixed income instruments of similar quality. If we find the yields on such securities with 1, 2, 3, 4 and 5 years maturities and plot such yields against their maturities, the resultant chart is an yield curve.
Normally the yield curves have positive slope i.e. there are lower yields for smaller maturities and higher yields for longer maturities. (This type of yield curve is referred as normal yield curve meaning that the yields rise as maturity lengthens). In the diagram given below, Curve (a) exhibits positive slope curve.

Sometimes, the curves have negative slope i.e. there are higher yields for smaller maturities and smaller yields for longer maturities. This type of yield curve is referred as inverted yield curve. In the diagram given below, Curve (c) exhibits negative slope curve.

Sometimes, the normal curves are almost flat i.e. yields do not vary much with the maturities. In the diagram given below, Curve (b) exhibits flat yield curve.

There are two important theories that explain the shape of the yield curve.

(i) expectation theory
(ii) liquidity preference theory
EXPECTATION THEORY: As per this theory, the shape of the yield curve depends on investors expectations. If they expect that in future the interest rates will rise, the curve will have positive slope; if they expect that in future the interest rates will fall, the curve will have negative slope. If the expectation is not much change in interest rates, the shape of the yield curve will be flat.

LIQUIDITY PREFERENCE THEORY: As per this theory, interest is reward for parting with the liquidity. This theory is based on the assumption that the investors have preference for liquidity; longer the maturity, more the moving away from the liquidity.

This theory provides the explanation only for normal yield curve. The theory states that for the longer maturities, the yields should be higher. Two reasons are advanced for this (i) longer maturities means more parting with the liquidity and (ii) longer maturities cause more risk and hence the higher risk premium.

FORWARD RATES

This concept is based on Expectation theory.

Forward rate is the interest rate that we expect, today, to prevail in the market after certain period.

Forward rate for first year is the rate of interest that expect, in the beginning of the 1st year, to earn on our investment made in the beginning of the 1st year till the end of 1st year. It is also referred as spot rate for 1st year.

Forward rate for second year is the interest rate that we expect, today, to prevail in the market in 2nd year. In another words, we can say that it is rate of return that we expect (today) to earn on our investment made in the beginning of 2nd year till the end of 2nd year.

Forward rate for 3rd year is the interest rate that we expect, today, to prevail in the market in 3rd year. In another words, we can say that it is rate of return that we expect (today) to earn on our investment made in the beginning of 3rd year till the end of 3rd year.

Forward rate for 4th year is the interest rate that we expect, today, to prevail in the market in 4th year. In another words, we can say that it is rate of return that we expect (today) to earn on our investment made in the beginning of 4th year till the end of 4th year.

And so on…………………..

We can calculate the forward interest rate either with the help of wealth ratio or using the basic concept of valuation (The basic concept of valuation is that the market price of any asset is equal to present value of all future cash flows).

Q. No. 23 : The following is the list of prices of zero coupon bonds of various maturities. Calculate the yields to maturity of each bond.
Maturity (years).                        Market price of Rs.1000 face value bond
1                                                 Rs.952.38
2                                                 Rs.890.00
3                                                 Rs.816.30

Answer
YTM
1 year :
Let YTM = r
952.38(1+r) = 1000
r = 5%

2 Years :
Let YTM = r
890(1+r)^2 = 1000
r = 6%

3 Years :
Let YTM = r
816.30(1+r)^3 = 1000
r = 7%

Q. No. 24 : The following table represents the yield curve of a particular types of bonds issued by a company:

<table>
<thead>
<tr>
<th>Maturity period (Years)</th>
<th>YTM %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

What are the forward rates?

Answer
Year 1: Current interest rate for 1 year (called as spot rate, also called as forward rate for year 1) = 10%

Year 2 : Let’s invest Rs.100. It will grow to Rs.110 at the end of 1st year and to 123.21 at the end of 2nd year.
Forward rate for 2nd year = [(123.21/110) -1] = 12.001%

Year 3: Let’s invest Rs.100. It will grow to Rs.110 at the end of 1st year and to 123.21 at the end of 2nd year and to 140.4928 at the end of 3rd year.
Forward rate for 3rd year = [(140.4928)/(123.21) -1] = 14.027%

Q. No. 25 : In an economy, the prices of Bonds reveal the following pattern of forward rates :

<table>
<thead>
<tr>
<th>Year</th>
<th>Forward rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7%</td>
</tr>
<tr>
<td>2</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>9%</td>
</tr>
</tbody>
</table>
Suppose you are interested in purchasing a 6% Bond of Rs.1000, maturity 3 years, what should be the price.

**Answer**

Price of the bond = 
\[
\frac{60}{(1.07)} + \frac{60}{(1.07)(1.08)} + \frac{1060}{(1.07)(1.08)(1.09)}
\]
\[
= 949.53
\]

Q. No. 26 : From the following data for Government securities, calculate the forward rates:

<table>
<thead>
<tr>
<th>Face value (Rupees)</th>
<th>Interest rate (%)</th>
<th>Maturity year(s)</th>
<th>Current price (Rupees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,00,000</td>
<td>0</td>
<td>1</td>
<td>91,500</td>
</tr>
<tr>
<td>1,00,000</td>
<td>10</td>
<td>2</td>
<td>98,500</td>
</tr>
<tr>
<td>1,00,000</td>
<td>10.50</td>
<td>3</td>
<td>99,000</td>
</tr>
</tbody>
</table>

(Nov. 2007)

**Answer**

Year 1: Current interest rate for 1 year (called as spot rate, also called as forward rate for year 1) = \( \frac{1,00,000}{91,500} - 1 = 9.29\% \)

Year 2: let forward rate for year 2 = r

\[
98,500 = \frac{10000}{1.0929} + \frac{110,000}{(1.0929)(1+r)}
\]

\[
r = 12.63\%
\]

Year 3: let the forward rate for year 3 = r

\[
99,000 = \frac{110500}{1.0929} + \frac{10,500}{(1.0929)(1.1263)} + \frac{1,10,500(1.0929)(1.1263)(1+r)}{1.0929(1.1263)}
\]

\[
r = 11.01\%
\]

Q. No. 27 : The YTM of 1-year maturity zero coupon bond is 6% and that of 2-year maturity zero coupon bond is 7%. If the company issues a 2-year maturity 8% coupon bond of Rs.1000 face value, what should be appropriate issue price?

**Answer**

Appropriate price = 80/(1.06) + 1080/(1.07)^2 = 1018.79

Alternative way:

Suppose we invest Rs.100 today, it will grow to Rs.106 after 1 year.

Suppose we invest Rs.100 today, it will grow to 100(1.07)^2 after 2 years.

Forward rate for year 1 = 6%

Forward rate for year 2 = \( \frac{100(1.07)^2 / 1.06 }{1.06 } - 1 = 8.009\% \)

Appropriate price = 80/(1.06) + 1080/((1.06)(1.08009)) = 1018.79
Q. No. 28: Below is a list of the prices of zero coupon bonds of various maturities issued by a company:

<table>
<thead>
<tr>
<th>Maturity [Year(s)]</th>
<th>Price of Rs.1000 face value bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>943.40</td>
</tr>
<tr>
<td>2</td>
<td>873.52</td>
</tr>
<tr>
<td>3</td>
<td>816.37</td>
</tr>
</tbody>
</table>

Find the forward rates. Suppose an 8.50% Rs.1000 face value 3-years maturity bond of a similar company is available in the market, what should be its appropriate price?

Answer

YTM
1 year:
Let YTM = r
943.40(1+r) = 1000
r = 6%

2 Years:
Let YTM = r
873.52(1+r)^2 = 1000
r = 7%

3 Years:
Let YTM = r
816.37(1+r)^3 = 1000
r = 7%

Appropriate price = \(\frac{85}{1.06} + \frac{85}{1.06(1.07)} + \frac{1085}{(1.06)(1.07)^3}\)
= 1040.11

Alternative way:
Suppose we invest Rs.100 today, it will grow to Rs.106 after 1 year.
Suppose we invest Rs.100 today, it will grow to 100(1.07)^2 after 2 years.
Suppose we invest Rs.100 today, it will grow to 100(1.07)^3 after 3 years.

Forward rate for year 1 = 6%
Forward rate for year 2 = \(\frac{100(1.07)^2}{1.06} - 1 = 8.009\%
Forward rate for year 3 = \(\frac{100(1.07)^3}{(1.07)^2} - 1 = 7.00\%

Appropriate price
= \(\frac{85}{1.06} + \frac{85}{(1.06)(1.08009)} + \frac{1085}{(1.06)(1.08009)(1.07)}\) ]
= 1040.12

Convexity
Limitations of Modified Duration:
Modified duration is a measure of rate of change in the bond price on change in yield to maturity. Suppose the modified duration is 2. It means for every 1 percentage point change in YTM, the bond value will change inversely by 2%. Suppose the current value of a bond is Rs.90 and YTM is 10%; if YTM increases to 11%, the value of bond will be Rs.88.20; if the YTM decreases to 8%, the value of the bond will be Rs.91.80.

Let us understand this point more clearly. Suppose (i) the current value of a bond is Rs.90 (ii) YTM is 10% and (iii) Modified duration is 1. The following table gives change in Bond price on various changes in YTMs.

<table>
<thead>
<tr>
<th>Change in YTM (in terms of basis points)</th>
<th>% change in bond price</th>
<th>Bond value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>+ 1%</td>
<td>90.90</td>
</tr>
<tr>
<td>+ 100</td>
<td>- 1%</td>
<td>89.10</td>
</tr>
<tr>
<td>- 200</td>
<td>+ 2%</td>
<td>91.80</td>
</tr>
<tr>
<td>+ 200</td>
<td>- 2%</td>
<td>88.20</td>
</tr>
<tr>
<td>-300</td>
<td>+ 3%</td>
<td>92.70</td>
</tr>
<tr>
<td>+300</td>
<td>- 3%</td>
<td>87.30</td>
</tr>
<tr>
<td>-400</td>
<td>+ 4%</td>
<td>93.60</td>
</tr>
<tr>
<td>+400</td>
<td>-4%</td>
<td>86.40</td>
</tr>
</tbody>
</table>

The above data concludes that the relationship between YTM and bond value is linear and inverse i.e. if we draw graph using the above data (taking YTM on y-axis and bond value on x-axis), we will obtain a downward moving straight line i.e. the graph of the relationship between price and YTM is in the form of a downward moving straight line.

The reality (with respect to linearity), however, is different i.e. the true relationship between a change in price of bond and a change in YTM is not linear. Duration provides an approximation of this relationship. For a small change in YTM, duration estimates the changed price more or less accurately, but for larger changes duration becomes less accurate.

Q. No. 29: 10 % bond with maturity 5 years. Face value Rs.100. Current YTM is 10%.
(a) (i) Find value of the bond.
(ii) What will be the value of the bond if the YTM increases from 10% to 11%?
(iii) What will be the value of the bond if the YTM decreases from 10% to 9%?
(iv) What will be the value of the bond if the YTM increases from 10% to 12%?
(v) What will be the value of the bond if the YTM decreases from 10% to 8%?

(b) Find modified duration. Using modified duration:
(i) What will be the value of the bond if the YTM increases from 10% to 11%?
(ii) What will be the value of the bond if the YTM decreases from 10% to 9%?
(iii) What will be the value of the bond if the YTM increases from 10% to 12%?
(iv) What will be the value of the bond if the YTM decreases from 10% to 8%?

Answer (a)

(i) Rs. 100
(ii) Value of bond = \(10 \times 3.6959 + 100 \times 0.59345\) = 96.30
(iii) Value of bond = \(10 \times 3.890 + 100 \times 0.650\) = 103.90
(iv) Value of bond = \(10 \times 3.605 + 100 \times 0.567\) = 92.75
(v) Value of bond = \(10 \times 3.993 + 100 \times 0.681\) = 108.03

Answer (b)

(b) Duration

<table>
<thead>
<tr>
<th>X</th>
<th>W</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.09</td>
<td>9.09</td>
</tr>
<tr>
<td>2</td>
<td>8.26</td>
<td>16.52</td>
</tr>
<tr>
<td>3</td>
<td>7.51</td>
<td>22.53</td>
</tr>
<tr>
<td>4</td>
<td>6.83</td>
<td>27.32</td>
</tr>
<tr>
<td>5</td>
<td>68.31</td>
<td>341.55</td>
</tr>
<tr>
<td>Total = 15</td>
<td>100</td>
<td>417.01</td>
</tr>
</tbody>
</table>

Duration = \(\frac{417.01}{100}\) = 4.1701

Modified duration = \(\frac{4.1701}{1.10}\) = 3.791

(i) Value of bond = \(100 - 3.791\) = 96.209
(ii) Value of bond = \(100 + 3.791\) = 103.791
(iii) Value of bond = \([100 - (2 \times 3.791)]\) = 92.418
(iv) Value of bond = \([100 + (2 \times 3.791)]\) = 107.582

Conclusion of above discussion:

Value of Bond is present value of all cash flows from that bond, the present value being calculated using YTM). If we calculate the bond values using this concept, we find that

(i) The increase in value for a decrease in YTM will be greater than the decrease in value due to an equal rise in YTM. (Suppose the YTM declines by 1% and as result the bond value increase by X%; Now suppose the YTM increases by 1% and a result the bond value decreases by Y%; comparing X with Y we find that X > Y.)
(ii) Change in the bond value on change in YTM is not linear i.e. if 1% change in YTM causes X change (in the opposite direction) in the bond value, the change in bond value on 2% change is not 2X.
Convexity is a better technique to find the change in the value of the bond on change in YTM. When we calculate change in bond value due to change in YTM using the convexity, values are nearer to the values obtained by direct calculations (By direct calculation we mean finding the value of bond as present value of all cash flows from that bond). Changes in bond values due to changes in YTM using convexity are

(i) non-linear (ii) upside capture, downside protection. (The increase in value for a decrease in YTM will be greater than the decrease in value due to an equal rise in YTM. Suppose the YTM declines by 1% and as result the bond value increase by x%; Now suppose the YTM increases by 1% and a result the bond value decreases by y%; comparing X with Y we find that X > Y.)

\[ \text{Convexity} = \frac{\sum XW(1+X)}{\text{Current Bond value}} \]

Where, X represents the series of periods of cash inflows, and W represents Present value (calculated on the basis of prevailing YTM) of cash inflows corresponding to X periods

Using Convexity:

\[ \% \text{ Change in the price of the bond on change in YTM:} \]
\[ = \left[ (-1) (\text{Modified Duration})(\text{Yield change in terms of percentage points}) \right] \]
\[ + \left[ \left( \frac{C}{200} \right)^2 \right] \]

**Q. No.30**: 10% bond with maturity 5 years. Face value : Rs.100. Current YTM = 10 %. Find convexity. Using convexity, estimate the value of bond assuming YTM at 8%; what if 12%.

**Answer**: Statement showing calculation of convexity:

<table>
<thead>
<tr>
<th>X</th>
<th>W (Present values of cash flows)</th>
<th>XW(1+ X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.09</td>
<td>9.09x2 = 18.18</td>
</tr>
<tr>
<td>2</td>
<td>8.26</td>
<td>16.52x3 = 49.56</td>
</tr>
<tr>
<td>3</td>
<td>7.51</td>
<td>22.53x4 = 90.12</td>
</tr>
<tr>
<td>4</td>
<td>6.83</td>
<td>27.32x5 = 136.60</td>
</tr>
<tr>
<td>5</td>
<td>68.31</td>
<td>341.55x6 = 2049.30</td>
</tr>
<tr>
<td>Total = 15</td>
<td></td>
<td>2343.76</td>
</tr>
</tbody>
</table>

Convexity = \( \frac{\sum XW(1+X)}{\text{Current Bond value}} \) = 2343.76/ 100 = 23.4376

Using Convexity:
% Change in the price of the bond if YTM increases from 10% to 12%:

\[
\begin{align*}
&= \left\{ (-1)(\text{Modified Duration})(\text{Yield change in terms of percentage points}) \right\} \\
&\quad + \frac{C}{200} (\text{Yield Chg in terms of percentage points})^2 \\
&= \left\{ (-1)(3.791)(2) \right\} + \frac{23.4376}{200}(4) \\
&= -7.582 + 0.47 = -7.112
\end{align*}
\]

% Change in bond price if YTM decreases from 10% to 8%:

\[
\begin{align*}
&= \left\{ (-1)(3.791)(-2) \right\} + \frac{23.4376}{200}(4) \\
&= 8.052
\end{align*}
\]

<table>
<thead>
<tr>
<th>YTM (%)</th>
<th>Bond value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As per PV method</td>
</tr>
<tr>
<td>8%</td>
<td>108.03</td>
</tr>
<tr>
<td>10%</td>
<td>100</td>
</tr>
<tr>
<td>12%</td>
<td>92.75</td>
</tr>
</tbody>
</table>

Q. No. 31: Refer to Q.No.29 and Q.No.30. Fill in the blanks of the following table.

<table>
<thead>
<tr>
<th>YTM (%)</th>
<th>Bond value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As per PV method</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Answer: % Change in the price of the bond if YTM increases from 10% to 11%:

\[
\begin{align*}
&= \left\{ (-1)(\text{Modified Duration})(\text{Yield change in terms of percentage points}) \right\} \\
&\quad + \frac{C}{200} (\text{Yield Chg in terms of percentage points})^2 \\
&= \left\{ (-1)(3.791)(1) \right\} + \frac{23.4376}{200}(1) \\
&= -3.6738
\end{align*}
\]

% Change in bond price if YTM decreases from 10% to 9%:

\[
\begin{align*}
&= \left\{ (-1)(3.791)(-1) \right\} + \frac{23.4376}{200}(1) \\
&= 3.9082
\end{align*}
\]
<table>
<thead>
<tr>
<th>YTM (%)</th>
<th>As per PV method</th>
<th>As per Mod. Duration method</th>
<th>As per convexity method</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>108.03</td>
<td>107.582</td>
<td>108.0520</td>
</tr>
<tr>
<td>9</td>
<td>103.90</td>
<td>103.791</td>
<td>103.9082</td>
</tr>
<tr>
<td>10</td>
<td>100.00</td>
<td>100.00</td>
<td>100.0000</td>
</tr>
<tr>
<td>11</td>
<td>96.30</td>
<td>96.209</td>
<td>96.3262</td>
</tr>
<tr>
<td>12</td>
<td>92.75</td>
<td>92.418</td>
<td>92.888</td>
</tr>
</tbody>
</table>
Q. No. 32: The following are the details about a 6% bond:
Market price: par  Maturity 10 years.  Face value = 1000

Find the duration of the bond. Find the convexity of the bond. Suppose the YTM rises to 7%, find the value of the bond (i) using duration (ii) using convexity (iii) without using duration or convexity. Comment on the results.

Answer:

<table>
<thead>
<tr>
<th>X</th>
<th>W</th>
<th>XW</th>
<th>XW(1+X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 x 0.943 =56.58</td>
<td>56.58</td>
<td>113.16</td>
</tr>
<tr>
<td>2</td>
<td>60 x 0.890 = 53.40</td>
<td>106.80</td>
<td>320.40</td>
</tr>
<tr>
<td>3</td>
<td>60 x 0.840 = 50.4</td>
<td>151.20</td>
<td>604.80</td>
</tr>
<tr>
<td>4</td>
<td>60 x 0.792 =47.52</td>
<td>190.08</td>
<td>950.40</td>
</tr>
<tr>
<td>5</td>
<td>60 x 0.747 = 44.82</td>
<td>224.10</td>
<td>1344.60</td>
</tr>
<tr>
<td>6</td>
<td>60 x 0.705 = 42.3</td>
<td>253.80</td>
<td>1776.60</td>
</tr>
<tr>
<td>7</td>
<td>60 x 0.665 = 39.9</td>
<td>279.30</td>
<td>2234.40</td>
</tr>
<tr>
<td>8</td>
<td>60 x 0.627 = 37.62</td>
<td>300.96</td>
<td>2708.64</td>
</tr>
<tr>
<td>9</td>
<td>60 x 0.592 = 35.52</td>
<td>319.68</td>
<td>3196.80</td>
</tr>
<tr>
<td>10</td>
<td>1060 x 0.558 = 591.48</td>
<td>5914.80</td>
<td>65062.80</td>
</tr>
<tr>
<td>Total</td>
<td>999.54*</td>
<td>7797.30</td>
<td>79312.60</td>
</tr>
</tbody>
</table>

*It should have been 1000. The difference is due to calculation approximations.

- Duration = 7797.30/999.54 = 7.80  Modified duration = 7.80/1.06 = 7.3585. If YTM rises to 7%, the bond value decreases by 7.3585%.
  New bond value = 926.42
- Convexity = 79312.60/999.54 = 79.35

% change in Bond price : % Change in the price of the bond if YTM increases from 6% to 7%:
\[\begin{align*}
&= \left(-1\right) \text{(Modified Duration)} \times \left(\text{Yield change in terms of percentage points}\right) \\
&\quad + \left(C/200\right) \times \left(\text{Yield Chg in terms of percentage points}\right)^2 \\
&= \left(-1\right) \times \left(7.3585\right) \times \left(1\right) + \left(79.35\right)/200 \times \left(1\right) \\
&= 6.96\% \\
\end{align*}\]

New Price of Bond = 1000 - 69.60 = 930.40

- Present value of cash flow method:

New price of bond (YTM 7%) = 60(7.024) + 1000 x 0.508 = 929.44

% Error by duration method = (926.42 - 929.44) / 929.44 = 0.325%

% Error by convexity method = (930.40 - 929.44) / 929.44 = 0.103%

**Comment**: The new price of bond (calculated on the basis of changed YTM) is more accurate when calculated by convexity (rather than by duration)

---

**CONVERTIBLE BONDS**

These are the bonds which have / can to be converted into specified number of equity shares of the company issuing these bonds within a specified period. In India most of the convertible bonds have been issued on the basis of compulsory conversion i.e. the bonds are compulsorily convertible into number of specified number of shares, there is no discretion of the bond holder. In USA and European countries, convertible bonds are option convertible bonds i.e. conversion takes place if the bond holder so desire.

An Example of option convertible bond: Suppose a company issues 7% convertible bonds of $100 each, maturity 7 years, redemption at par. The bondholder can get his bond converted into 4 equity shares after 2 years of issuance. Now whether the bond will be converted into equity shares or not, it is at the discretion of the bond holder. Suppose he gets the bonds converted into shares, the company’s liability towards principal and interest will extinguish. If he does not get the bond converted, he will be emitted to receive interest periodically and on maturity he will get the redemption amount of $100.

The convertibility option lowers the interest rate that the issuer would otherwise have to pay without this feature, and it appeals to investors who want current income, but would like to take advantage of any growth in the issuer company.

Let’s understand a few terms related to convertible debentures:

(i) The number of shares that each bond can be converted to is known as the conversion ratio.

(ii) Conversion price is the exercise price at which the investor converts his bond into equity shares. It is obtained by dividing the par value of the bond by the conversion ratio.
The term Conversion parity price is obtained by dividing the current market price of convertible debenture by conversion ratio. For instance, if the current price of the bond is Rs. 1500 and it can be converted to 10 equity shares, the conversion parity price is Rs.150. Suppose an investor buys a convertible debenture from the Stock – Market, immediately converts the into equity shares and sells these shares, he shall breakeven if the ruling market price of the share is equal to conversion parity price.

(iii) Straight value of a Bond : This term is used with reference to option convertible bonds. The straight bond value is what the convertible bond would sell for if it could not be converted into equity shares. It is the price of an equivalent non-convertible bond.

(iv) Downside risk : If the share price goes much below the conversion price, it is expected by market forces that conversion option won’t be exercised, the market price of the convertible bond will be equal to Straight value of bond. This will result in loss for the investor. This loss is referred as downside risk. If to be calculated in % terms, this amount should be divided by straight value of bond and multiplied by 100.

(vi) Stock value of bond = current market price of share x conversion ratio. It is also referred as conversion value of the bond.

(vii) Conversion premium The extent by which the market value of a convertible security exceeds the conversion value. Suppose a convertible bond is being traded in the market at Rs. 265. It can be converted into 10 equity shares having market price of Rs.25. The conversion premium is 265 – 250 = 15. If calculated in terms of %, the conversion premium is divided by conversion value.

Q. No.33 The data given below relates to a convertible bond:
Face value : Rs.250
Coupon rate : 12%
No. of shares per bond :20
Market price of share : Rs.12
Straight value of bond : Rs.235
Market price of convertible bond : Rs.265
Calculate:
(i) Stock value of bond.
(ii) The percentage of downside risk. (iii) The conversion premium
(iv) The conversion parity price of the stock. **(Nov. 2008 SFM)**
Answer
(i) Stock value of bond = current market price of share x conversion ratio = 12 x 20 = 240
(ii) % of downward risk = 
\[
\frac{\text{Market price of convertible bond} - \text{straight value of convertible bond}}{\text{Straight value of bond}}
\]
\[
= \frac{(265 - 235)}{235} = 0.1277 = 12.77\%
\]
(iii) Conversion premium = current market price of convertible bond – conversion value = 265 – 240 = 25
(iv) Conversion parity price
\[
= \frac{\text{current market price of convertible debenture}}{\text{conversion ratio}}.
\]
\[
= \frac{265}{20} = 13.25
\]

Interest Immunization

We know that investment in the bonds is subject to three risks (i) Default risk and (ii) interest risk and (iii) reinvestment rate risk. The first one is referred as unsystematic risk while the other two are referred as systematic risks. Immunization is a strategy that takes care of systematic risk. It ensures that a change in interest rate will not affect the expected return from a bond – portfolio. Change in interest rates affects the return from the bonds investment in two ways (i) there is change in the value of the bond and (ii) change in the income from the reinvestment. Changes in interest rates have opposite effects of change in bond values and that in reinvestment incomes. For example, an increase in interest rates hurts the bond value; it helps by increasing the return from the reinvestments and vice-versa. Immunization aims at offsetting the effects of the two changes so that the investor’s total return remains constant regardless of whether there is rise or fall in the interest rates.

A portfolio is immunized when its duration (average duration of the bonds constituting the portfolio; weights being the amounts of the investments in different bonds) equals the investor’s time horizon. In other words, if the average duration of portfolio equals the investor’s planned investment period, the realized return equals to the expected return.

Q No.34: Mr. X has to pay Rs.10,000 after 5 years from today. He wants to fund this obligation today only. On enquiry he gathers that a company has come out with a initial public offer of 8% Bonds (face value Rs.100) maturity 6 years. Interest on such bonds is payable annually. He invests Rs.6800 in the offer. What amount he will accumulate if market interest rate continues to be 8%. What if the market interest falls to 6% or rises to 10% immediately after his investment in the bonds. Do you have any offer to comment these amounts?
**Answer:**

If market interest continues to be 8%:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>544</td>
<td>544(1.08)^4 = 740</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>544</td>
<td>544(1.08)^3 = 685</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>544</td>
<td>544(1.08)^2 = 635</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>544</td>
<td>544(1.08)^1 = 588</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>544</td>
<td>544(1.08)^0 = 544</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>---</td>
<td>6,800</td>
<td>6,800</td>
</tr>
</tbody>
</table>

Total Rs.9,992

If market interest immediately declines to 6%:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>544</td>
<td>544(1.06)^4 = 687</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>544</td>
<td>544(1.06)^3 = 648</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>544</td>
<td>544(1.06)^2 = 611</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>544</td>
<td>544(1.06)^1 = 577</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>544</td>
<td>544(1.06)^0 = 544</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>---</td>
<td>(6800+544)/1.06 = 6,928</td>
<td></td>
</tr>
</tbody>
</table>

Total Rs.9,995

If market interest immediately rises to 10%:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>544</td>
<td>544(1.10)^4 = 796</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>544</td>
<td>544(1.10)^3 = 724</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>544</td>
<td>544(1.10)^2 = 658</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>544</td>
<td>544(1.10)^1 = 598</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>544</td>
<td>544(1.10)^0 = 544</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>---</td>
<td>(6800+544)/1.10 = 6,676</td>
<td></td>
</tr>
</tbody>
</table>

Total Rs.9,996

In all the three cases, the total realization is almost the same (there is negligible difference due to calculations approximations). This is possible only in one case i.e. the duration of the bond investment matches with the investor’s time horizon, i.e. the duration is 5. Let’s check the duration of this bond investment:

<table>
<thead>
<tr>
<th>X</th>
<th>PV of cash inflows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>544 x 0.926 = 503.74</td>
<td>1 x 503.74</td>
</tr>
<tr>
<td>2</td>
<td>544 x 0.857 = 466.21</td>
<td>2 x 466.21</td>
</tr>
<tr>
<td>3</td>
<td>544 x 0.794 = 431.94</td>
<td>3 x 431.94</td>
</tr>
<tr>
<td>4</td>
<td>544 x 0.735 = 399.84</td>
<td>4 x 399.84</td>
</tr>
<tr>
<td>5</td>
<td>544 x 0.681 = 370.46</td>
<td>5 x 370.46</td>
</tr>
<tr>
<td>6</td>
<td>7344 x 0.630 = 4626.72</td>
<td>6 x 4626.72</td>
</tr>
<tr>
<td>ΣW=6,799</td>
<td></td>
<td>ΣXW=34,064</td>
</tr>
</tbody>
</table>
Duration = ∑XW / ∑W = 34,064 / 6,799 = 5.01 (it is as good as 5)

The investor’s time horizon matches with the duration of the bond. Hence, the change in the market rates could not change the return; in other words, the bond investment remains immunized against the interest rate risk (also known as systematic risk) as the duration of the bond investment matched with investor’s time horizon.

Q. No. 35: Find the duration of an irredeemable bond.
Answer: Let the face value of irredeemable bond is Rs.10 and it carries 10% coupon.

<table>
<thead>
<tr>
<th>X</th>
<th>W</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1. (1/1+ YTM)</td>
<td>1 x 1. (1/1+ YTM)</td>
</tr>
<tr>
<td>2</td>
<td>1. (1/1+ YTM)²</td>
<td>2 x 1. (1/1+ YTM)²</td>
</tr>
<tr>
<td>3</td>
<td>1. (1/1+ YTM)³</td>
<td>3 x 1. (1/1+ YTM)³</td>
</tr>
<tr>
<td>4</td>
<td>1. (1/1+ YTM)⁴</td>
<td>4 x 1. (1/1+ YTM)⁴</td>
</tr>
<tr>
<td>5</td>
<td>........................</td>
<td>..................</td>
</tr>
<tr>
<td>6</td>
<td>........................</td>
<td>..................</td>
</tr>
<tr>
<td>7</td>
<td>........................</td>
<td>..................</td>
</tr>
<tr>
<td>8</td>
<td>........................</td>
<td>..................</td>
</tr>
<tr>
<td>And so on</td>
<td>........................</td>
<td>..................</td>
</tr>
</tbody>
</table>

∑W = B                              ∑XW = A

Duration = ∑XW / ∑W = A

Where B = (1/1+ YTM) + (1/1+ YTM)² + (1/1+ YTM)³ + (1/1+ YTM)⁴ + ...... = 1/YTM

Where A =

= (1/1+ YTM) + 2.(1/1+ YTM)² + 3.(1/1+ YTM)³ + 4.(1/1+ YTM)⁴ + ......
= (1/1+ YTM) + (1/1+ YTM)² + (1/1+ YTM)³ + (1/1+ YTM)⁴ + ......
= (1/1+ YTM) + (1/1+ YTM)² + (1/1+ YTM)³ + (1/1+ YTM)⁴ + ......
= (1/1+ YTM) + (1/1+ YTM)² + (1/1+ YTM)³ + (1/1+ YTM)⁴ + ......
= [1/YTM] + [1/YTM(1+ YTM)] + 1/YTM(1+ YTM)² + 1/YTM(1+ YTM)³ + .........

= [(1/YTM) x ((1) + (1/YTM) + (1/1+ YTM)² + (1/1+ YTM)³ + ......}]

= (1/YTM) x {(1) + (1/YTM)}

= (1/YTM) x (YTM + 1)/YTM
equal to \((YTM + 1) / (YTM)^2\)

Duration = \(A/B = [(YTM + 1)/(YTM)^2] / (1/YTM) = (1+YTM)/YTM\)

**Q. No. 36:** A company has to pay Rs.12411 after 6 years from today. Current market interest rate is 10%. It wants to fund this obligation today only. The following two bonds provide 10% return:

- (1) Zero coupon bond maturity : 4 years
- (2) 10% Irredeemable bond

Suggest bond portfolio which is immunized against the systematic risk. What amount you will receive at the end of 6 years by redeeming this portfolio. What if the current market interest changes to 11% at the end of 2\(^{nd}\) year.

**Answer:**

Duration of zero coupon bond is 4 and that of irredeemable bond is 11. The investment for funding the obligation should have duration of 6 for being immunized against interest rate change. Let’s invest \(W_1\) in zero coupon bond and \(1-W_1\) in irredeemable bond.

\[(W_1 x 4) + (1 - W_1) x 11 = 6,\]

Solving the equation we get, \(W_1 = 5/7\)

Present value of the obligation = \(12411 / (1.10)^6 = Rs.7,000\)

The company may invest Rs. 5000 in zero coupon bond (Maturity value Rs.7321) and Rs.2000 in Irredeemable bond.

If market interest continues to be 10%, the investment will fetch Rs.12411 (assuming that all intermediary cash inflows will be invested at 10% p.a. without any loss of time)

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>200(1.10)(1.11)^1 = 334</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>200 (1.11)^2 = 304</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>200 (1.11)^3 = 274</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>200 (1.11)^4 = 246</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7321</td>
<td>7321(1.11)^2 = 9020</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>200(1.11)^3 = 222</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>200.00 = 200</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1,818</td>
<td>1818.00 = 1818</td>
<td></td>
</tr>
</tbody>
</table>

\(\text{Rs.12418}\)
Q. No. 37: An 18-years maturity 10% bond is traded in the market at par. Another 9-years maturity 10% bond is also traded at par. An investor’s time horizon is 9 years and his target return of 10%. Which bond he should opt for investment? Find his accumulated wealth after 9 years (assuming that the interest received is immediately reinvested) inn each of the following three situations:
(i) YTM continues to be 10% for the entire period
(ii) YTM drops to 9% immediately after the investment is made
(iii) YTM rises to 10% immediately after the investment is made.
Comment on immunization of interest rate change.

Answer:
Suppose the par value is Rs.1000.
If the YTM continues at 10% : Wealth at the end of 9 years period would, in either case, be 1000(1.10)⁹ = 2358.50
If market interest immediately rises to 11%: (9 years Maturity)

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100(1.11)⁸</td>
<td>230.45</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100(1.11)⁷</td>
<td>207.62</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100(1.11)⁶</td>
<td>187.04</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100(1.11)⁵</td>
<td>168.51</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100(1.11)⁴</td>
<td>151.81</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>100(1.11)³</td>
<td>136.76</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>100(1.11)²</td>
<td>123.21</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>100(1.11)¹</td>
<td>111.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100(1.11)⁰</td>
<td>100.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>1000</td>
<td>1000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2416.30</td>
</tr>
</tbody>
</table>

If market interest immediately falls to 9%: (9 years Maturity)

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100(1.09)⁸</td>
<td>199.26</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100(1.09)⁷</td>
<td>182.80</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100(1.09)⁶</td>
<td>167.10</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100(1.09)⁵</td>
<td>153.87</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100(1.09)⁴</td>
<td>141.16</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>100(1.09)³</td>
<td>129.50</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>100(1.09)²</td>
<td>118.81</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>100(1.09)¹</td>
<td>109.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100(1.09)⁰</td>
<td>100.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>1000.00</td>
<td>1000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2301.50</td>
</tr>
</tbody>
</table>
The difference between the amounts of targeted accumulated wealth and actual wealth is substantial. It means in this case the target return has not been achieved if the interest rate changed. It means the investment is not immunized against the interest rate change.

If market interest immediately rises to 11%: (18 years Maturity)

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100(1.11)^8</td>
<td>230.45</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100(1.11)^7</td>
<td>207.62</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100(1.11)^6</td>
<td>187.04</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100(1.11)^5</td>
<td>168.51</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100(1.11)^4</td>
<td>151.81</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>100(1.11)^3</td>
<td>136.76</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>100(1.11)^2</td>
<td>123.21</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>100(1.11)^1</td>
<td>111.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100(1.11)^0</td>
<td>100.00</td>
</tr>
<tr>
<td>9</td>
<td>–</td>
<td>944.70</td>
<td>944.70</td>
</tr>
</tbody>
</table>

The difference between the amounts of targeted accumulated wealth and actual wealth is just negligible. It means in this case the target return has been achieved even if the interest rate changed.

If market interest immediately falls to 9%: (18 years Maturity)

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100(1.09)^8</td>
<td>199.26</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100(1.09)^7</td>
<td>182.80</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100(1.09)^6</td>
<td>167.10</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100(1.09)^5</td>
<td>153.87</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100(1.09)^4</td>
<td>141.16</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>100(1.09)^3</td>
<td>129.50</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>100(1.09)^2</td>
<td>118.81</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>100(1.09)^1</td>
<td>109.00</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100(1.09)^0</td>
<td>100.00</td>
</tr>
<tr>
<td>9</td>
<td>–</td>
<td>1059.50</td>
<td>1059.50</td>
</tr>
</tbody>
</table>

The difference between the amounts of targeted accumulated wealth and actual wealth is just negligible. It means in this case the target return has been achieved even if the interest rate changed.

This shows that by investing in 18 years maturity bonds, the investor remained immunized against interest rate risk (also known as systematic risk). This is possible only if the duration of the investment is equal to the time horizon of the investor i.e. 9 years.
To achieve the target return, he should invest in 18 years maturity bonds. This investment will provide him his target return in all cases (i) interest rate remains stable (ii) interest rate rises (iii) interest rate falls.

**Q. No. 38:** A company has to pay Rs.10m after 6 years from today. The company wants to fund this obligation today only. The current interest rate in the market is 8%. Two zero-coupon bonds are traded in the market on the basis of 8% YTM (a) maturity 5 years and (b) maturity 7 years. Suggest the interest rate risk immunized investment plan. Calculate the total amount to be received from the investments in following three cases: (i) market interest remains unchanged throughout the period of 6 years (ii) market interest rate declines to 6% 2 years after the investment was made (iii) market interest rate rises to 10% immediately after the investment was made.

**Answer:** Amount of investment to fund the obligation: \( \frac{10,000,000}{(1.08)^6} = Rs.6,301,696 \)

Investor’s time horizon is 6 years. The duration of the bond portfolio should be 6. Duration of 5 years maturity bond is 5 and that of 7 years maturity is 7. Let’s invest \( W_1 \) in 5 years maturity bonds and the balance of the investment amount in 7 years maturity bond.

\[
W_1(5) + (1- W_1)(7) = 6
\]

\[
W_1 = 0.50
\]

For interest rate immunization, the investor should invest Rs.3,150,848m in 5 years maturity bonds and Rs.3,150,848m in 7 years maturity bonds.

<table>
<thead>
<tr>
<th>Bond</th>
<th>Maturity amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years maturity</td>
<td>Rs. 4,629,629m</td>
</tr>
<tr>
<td>7 years maturity</td>
<td>Rs. 5,40m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.629629m</td>
<td>4.629629m(1.08) = 5m</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.000000m</td>
<td>= 5m</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>10m</td>
</tr>
</tbody>
</table>

If market interest continues to be 8%:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds</th>
<th>Accumulated amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.629629m</td>
<td>4.629629m(1.06) = 4.907407m</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.094400m</td>
<td>5.094400m = 5.09440m</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>10.001807m</td>
</tr>
</tbody>
</table>
If market interest rises:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest</th>
<th>Value of bonds (Rs. 4.629629m)</th>
<th>Accumulated amount (Rs. 4.909090m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td>Rs. 5.092592m</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Rs. 4.909090m</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>Rs. 10.001682m</td>
<td></td>
</tr>
</tbody>
</table>

Q. No. 39: A company has to pay Rs.1m at the end of each year for 3 years. The company wants to fund this obligation today only when current interest rate in the market is 10%. Two zero coupon bonds are being traded in the market (i) 1 years maturity and (ii) 5 years maturity. Suggest the interest rate risk immunized investment plan.

**Answer:** Let’s calculate the duration of the liability:

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of Cash flow (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rs.1m x 0.909</td>
<td>0.909m</td>
</tr>
<tr>
<td>2</td>
<td>Rs.1m x 0.826</td>
<td>1.652m</td>
</tr>
<tr>
<td>3</td>
<td>Rs.1m x 0.751</td>
<td>2.253m</td>
</tr>
<tr>
<td>Total</td>
<td>2.486</td>
<td>4.814</td>
</tr>
</tbody>
</table>

Duration of the liability: 4.814/2.486 = 1.94
Duration of 1 years maturity zero coupon bond = 1
Duration of 5 years maturity zero coupon bond = 5
Total investment to be made today = Rs.2.486m
Investment should be made in the two bonds in such a ratio that the duration of the bond portfolio is 1.94.

Let’s invest $W_1$ in 1 years maturity bonds and the balance of the investment amount in 5 years maturity bond.

\[ W_1(1) + (1-W_1)(5) = 1.94 \]

\[ W_1 = 0.765 \]

Investment in 1 year maturity : $0.765 \times 2.486 = 1.90179m$

Invest in 5 years maturity : $0.235 \times 2.486 = 0.58421m$

**EXTRA PRACTICE (MUST DO)**

**Convertible preference shares**

Q. No. 40 XYZ company has current earnings of Rs.3 per share with 5,00,000 shares outstanding. The company plans to issue 40,000 7% convertible preference shares Rs.50 each at par. The preference share is convertible in 2 equity shares. The current market price of the equity share is Rs.21.

(a) What is the preference shares’ conversion value?
(b) What is its conversion premium?
(c) Assuming that total earnings remain the same, calculate the effect of the issue on primary earning per share (i) before conversion and (ii) on a fully diluted basis.
(d) If profit after taxes increase by Rs. 1m, what will be primary earning per share (i) before conversion and (ii) on a fully diluted basis. (Nov. 2009 SFM)

Answer:
(a) Conversion value = current market price x conversion ratio = 21 x 2 = 42
(b) Conversion premium = current market price of convertible preference share - conversion value = 50 - 42 = 8

% conversion premium = (8/42) x 100 = 19.05%
(c) EPS before conversion:

<table>
<thead>
<tr>
<th>EAT</th>
<th>15,00,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference Dividend</td>
<td>1,40,000</td>
</tr>
<tr>
<td>Earnings available to equity shareholders</td>
<td>13,60,000</td>
</tr>
<tr>
<td>No. of equity shares</td>
<td>5,00,000</td>
</tr>
<tr>
<td>EPS</td>
<td>2.72</td>
</tr>
</tbody>
</table>

EPS after conversion:

<table>
<thead>
<tr>
<th>EAT</th>
<th>15,00,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings available to equity shareholders</td>
<td>15,00,000</td>
</tr>
<tr>
<td>No. of equity shares</td>
<td>5,00,000 + 80,000 = 5,80,000</td>
</tr>
<tr>
<td>EPS</td>
<td>2.59</td>
</tr>
</tbody>
</table>

(d) EPS before conversion:

<table>
<thead>
<tr>
<th>EAT</th>
<th>15,00,000 + 10,00,000 = 25,00,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference Dividend</td>
<td>1,40,000</td>
</tr>
<tr>
<td>Earnings available to equity shareholders</td>
<td>23,60,000</td>
</tr>
<tr>
<td>No. of equity shares</td>
<td>5,00,000</td>
</tr>
<tr>
<td>EPS</td>
<td>4.72</td>
</tr>
</tbody>
</table>

EPS before conversion:

<table>
<thead>
<tr>
<th>EAT</th>
<th>15,00,000 + 10,00,000 = 25,00,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings available to equity shareholders</td>
<td>25,00,000</td>
</tr>
<tr>
<td>No. of equity shares</td>
<td>5,00,000 + 80,000 = 580,000</td>
</tr>
<tr>
<td>EPS</td>
<td>4.31</td>
</tr>
</tbody>
</table>
Q. No. 41 Consider a 10% bond having maturity of 4 years and face value of Rs.1000. The interest rates in the market are expected to be 8% in year 1, 9% in year 2, 11% in year 3 and 12% in year 4. Current Price? YTM?

Answer:
Current Price = 100(0.926) + 100(0.926)(0.917) + 100(0.926)(0.917)(0.901) + 1100(0.926)(0.917)(0.901)(0.893) = 1005.56

Average return till Maturity = [(400 - 5.56)/4] / 1005.56 = 9.8064

NPV at 10%
-1005.56
+ 100(0.909 + 0.826 + 0.751 + 0.683)
+ 1000(0.683) = -5.66

NPV at 9%
-1005.56
+ 100(0.917 + 0.842 + 0.772 + 0.708)
+ 1000(0.708) = + 26.34

YTM:

<table>
<thead>
<tr>
<th>Lower rate</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower rate + --------------------------- x (difference in rates)</td>
<td></td>
</tr>
<tr>
<td>(Lower rate NPV - Higher rate NPV)</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{26.34}{26.34 - (-5.66)} = 9 + \frac{1}{8.82} \times 1 = 9.82\%
\]

Q. No. 42 Consider a 20% debenture that pays interest half yearly. What is effective annual interest rate?

Answer:
Effective annual interest rate means annually compounded interest rate. In this case, it is:

\[
100(1.10)(1.10) - 100 = 21\%
\]

Q. No. 43 Compare the effective annual interest rates of the following two securities:
(a) a 3 months maturity treasury bill, face value Rs.1000, currently selling at Rs.973.
(b) a 10 year maturity government security selling at par; coupon rate is 10% payable half-yearly.

Answer (a)
Interest rate for 3 months \(= \frac{27}{973} = 0.02775 = 2.775\%\)

Effective annual interest rate = 
\[
100 \times (1.02775)^4 - 100 = 11.57\%
\]

\(\textbf{b)}\) Effective annual interest rate \(= 100 \times (1.05)^4 - 100 = 10.25\%

**Q. N0.44** Daau ji Ltd issues 1m convertible debentures of Rs.100 each, maturity 10 years, annual coupon rate 10%. The debentures can be converted into 5 equity shares of the company at any time after the expiry of one year. Currently, the shares are quoted in the market at Rs.18 with PE ratio of 10: this ratio is likely to be maintained for years to come. The EPS of the company is expected to grow 10% p.a.

The company can call back the debentures following the international norms which are as follows:

“It is customary in most issues today, that the bonds should not be ‘callable’ for three years from the date of issue after which they will only be callable if issuer’s share price has risen over the conversion price, say by 30 per cent (or more) and has remained above such a level for a minimum period of time (normally 30 consecutive trading days).”

The company follows a policy of calling the debentures only in the beginning of the year. After how many years the debentures can be called back?

**Answer**
Conversion price is Rs.20. The debentures can be called if the share price reaches Rs.26 and remain at that level for 30 consecutive trading days.
Suppose today is 1.1.2008.

<table>
<thead>
<tr>
<th>Date</th>
<th>Market price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.2009</td>
<td>19.80</td>
</tr>
<tr>
<td>1.1.2010</td>
<td>21.78</td>
</tr>
<tr>
<td>1.1.2011</td>
<td>23.958</td>
</tr>
<tr>
<td>1.1.2012</td>
<td>26.3538</td>
</tr>
</tbody>
</table>

Assumption: 30 trading days are there in 6 weeks time.

Increase in price per week in 2011 \(= \left[ \frac{26.3538 - 23.958}{52} \right] = 0.04507\)
Increase in 46 weeks \(= 2.11936\)
Price on completion of 46 weeks of 2011 \(= 23.958 + 2.11936 = 26.07\)
It means during the last 6 weeks of 2011, the price in the market will be above Rs.26.

The debentures can be called on 1.1.2012 i.e. the debentures can be called after 4 years of the issue.
Q. No. 45 The following is the yield structure of AAA rated debenture :

<table>
<thead>
<tr>
<th>Period</th>
<th>Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>8.50</td>
</tr>
<tr>
<td>6 months</td>
<td>9.25</td>
</tr>
<tr>
<td>1 year</td>
<td>10.50</td>
</tr>
<tr>
<td>2 years</td>
<td>11.25</td>
</tr>
<tr>
<td>3 years and above</td>
<td>12.00</td>
</tr>
</tbody>
</table>

(a) Based on the expectation theory, calculate the implicit one-year forward rates in year 2 and year 3.

(b) if the interest rate increases by 50 basis points, what will be the percentage change in the price of the bond having a maturity of 5 years? Assume that the bond is fairly priced at the moment at Rs.1000. (Nov 2008)

Answer (a)

\[
\text{Forward rate for year 2} = \frac{1(1.1125)(1.1125)}{1(1.1050)} - 1 = 0.120051 = 12 \%
\]

\[
\text{Forward rate for year 3} = \frac{1(1.12)(1.12)(1.12)}{1(1.1125)(1.1125)} - 1 = 13.51\%
\]

Answer (b) Assumption: face value of the bond is Rs.1000.

Current interest (YTM) for 5 years maturity bond is 12%. The bond is priced at Rs.1000 i.e. the price of the bond is equal to face value. This is possible only when the coupon rate is equal to YTM (current interest rate). It means the annual interest on bond is Rs.120.

Price of the bond if interest rate increases by 0.50% :

\[
= 120 \times (\text{annuity at 12.50\% for five years}) + 1000\times (\text{PVF}_5 \text{ at 12.50\%})
\]

\[
= 120 \times (3.561) + 1000 \times (0.555) = 980
\]

Change in bond price = Rs.20

\%

\[
\text{change in bond price} = \frac{20}{100} \times 100 = 2\%
\]

Q. No.46: XL Ispat Ltd. has made an issue of 14 per cent non-convertible debentures on January 1, 2007. These debentures have a face value of Rs.100 and is currently traded in the market at a price of Rs.90. Interest on these NCDs will be paid through post-dated cheques dated June 30 and December 31. Interest payments for the first 3 years will be paid in advance through post-dated cheques while for the last 2 years post-dated cheques will
be issued at the third year. The bond is redeemable at par on December 31, 2011 at the end of 5 years. Required:
(i) Estimate the current yield and the YTM of the bond.
(ii) Calculate the duration of the NCD.
(iii) Assuming that intermediate coupon payments are not available for reinvestment calculate the realised yield on the NCD. (Nov. 2008)

**Answer**

(i) Current yield (per half year) = \( \frac{80}{90} \times 100 = 8.8889 \)%

i.e. 8.8889 \times 2 i.e. 17.7778 % p.a. half-yearly compounded.

Calculation of YTM:

Approximately rate of return = \( \frac{80}{5}/(90) \times 100 = 17.78 \) p.a.

Let YTM is 18% p.a. compounded half-yearly.

Value of bond =

\[ 7(\text{annuity for 10 periods at 9%}) + 100(\text{PVF of period 10 at 9%}) \]
\[ = 7(6.418) + 100(0.422) = 87.13 \]

NPV at 18% (compounded half yearly) : 87.13 – 90 = - 2.87

Let YTM is 17% p.a. compounded half-yearly.

Value of bond = \( \frac{7}{(1.085)^1} + \frac{7}{(1.085)^2} + \cdots + \frac{100}{(1.085)^{10}} \)

\[ = 90.16 \]

NPV at 17% (compounded half yearly) : 90.16 – 90 = 0.16

YTM (half-yearly) = 8.50 + \( \frac{0.16}{-2.87} \times 0.50 = 8.5264 \)

YTM = 8.5264 \times 2 = 17.05% p.a. half yearly compounded.

(ii) **Calculation of Duration**:

<table>
<thead>
<tr>
<th>X (Half-year)</th>
<th>( W )</th>
<th>( W \times X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 x 0.9214</td>
<td>6.4498</td>
</tr>
<tr>
<td>2</td>
<td>7 x 0.8491</td>
<td>11.8874</td>
</tr>
<tr>
<td>3</td>
<td>7 x 0.7824</td>
<td>16.4304</td>
</tr>
<tr>
<td>4</td>
<td>7 x 0.7209</td>
<td>20.1852</td>
</tr>
<tr>
<td>5</td>
<td>7 x 0.6643</td>
<td>23.2505</td>
</tr>
<tr>
<td>6</td>
<td>7 x 0.6121</td>
<td>25.7082</td>
</tr>
<tr>
<td>7</td>
<td>7 x 0.5640</td>
<td>27.6360</td>
</tr>
<tr>
<td>8</td>
<td>7 x 0.5197</td>
<td>29.1032</td>
</tr>
<tr>
<td>9</td>
<td>7 x 0.4789</td>
<td>30.1707</td>
</tr>
<tr>
<td>10</td>
<td>7 x 0.4413</td>
<td>30.8910</td>
</tr>
<tr>
<td>10</td>
<td>100 x 0.4413</td>
<td>441.3000</td>
</tr>
</tbody>
</table>

\[ \Sigma W = 90 \]

\[ \Sigma (W \times X) = 663.0124 \]
(iii) Let Realized yield for 1 rupee = x per half year
\[90(1+x)^{10} = 100 + 70\]
\[(1+x)^{10} = (100 + 70) / 90 = 1.889\]
\[\log(1+x)^{10} = \log 1.889\]
\[\log(1+x) = 0.2763/10 = 0.02763 \text{ (taking Anti-log)}\]
\[1+x = 1.065\]
\[x = 0.065 = 6.50\% \text{ per half year.}\]
(13\% p.a. half yearly compounded)

Alternative solution:
Let Realized yield for 1 rupee = x per year
\[90(1+x)^{5} = 100 + 70\]
\[(1+x)^{5} = (100 + 70) / 90 = 1.889\]
\[\log(1+x)^{5} = \log 1.889\]
\[\log(1+x) = 0.2763/5 = 0.05526 \text{ (taking Anti-log)}\]
\[1+x = 1.138\]
\[x = 0.1380 = 13.80 \text{ p.a. (annual compounded)}\]

Q.No.50 Today is 1.1.2009. Consider a bond: Face value Rs.100. YTM 10%. Coupon 8%. Coupon is payable on 31st December every year. What is the market price today? On 1.1.2010, will the market price be higher, lower or unchanged?

Answer: Assumption: the debentures are irredeemable.
MP Today : 80. MP on 1.1.2010 = Rs.80. The market price remains unchanged.

Q. No.47 A company issues a 10 years maturity zero coupon bond having face value (maturity value) of Rs.1000 on the basis of YTM of 10%. What is the imputed interest income in the first year, second year and 10th year?

Answer:
Let the issue price = x
\[x.(1.10)^{10} = 1,000\]
\[x = 385.55\]

Imputed interest at the end of I year = 38.56. Value at the end of I year = 424.11
Imputed interest at the end of II year = 42.41. Value at the end of II year = 466.52

Let the value in the beginning of 10th year = y
\[y(1.10) = 1000\]
\[y = 909.09\]
Imputed Interest for 10th year = 1000 - 909.09 = 90.91
Q. 48 A 10 years maturity bond redeemable at par has current yield of 8% and YTM of 9%. Is the bond selling above or below face value?

Answer:
(The concept of Current yield implies no capital gain/ no capital loss. We are purchasing the bonds for short period; we shall earn coupon rate based interest. After holding for short period, we shall sell at the rate we have purchased.)

YTM is 9% while current yield is 8%. It means if the bond is held till maturity, the capital gain is there. This is possible only if the bond is selling below face value.

Q. No.49 : A 5-years maturity bond carries coupon of 10%. The current interest rate in the market is 12%. Suppose you buy this bond today and sell after 1 year. What is the rate of return? Divide this return in two parts (i) interest and (ii) capital gain.

Answer :
YTM = 12%. Assumption : face value Rs.100
MP = 10(3.605) + 100(0.567) = 92.75
Rate of return in 1 year = 12%
Return in 1 year ( Per Bond) = 11.13
Interest in 1 year ( per Bond) = 10.00
Interest yield = 10/92.75  = 10.78%
Capital gain in 1 year ( per Bond) = 1.13
Capital gain yield = 1.13/92.75  = 1.22%

EXTRA PRACTICE QUESTIONS   (Optional)

Q. No. 50 : A 20 years maturity zero coupon bond is quoted in the market at Rs.29.334. Its maturity value is Rs.100. Find the yield to maturity.

Answer
Present value of Rs.100 to be received after 20 years = Rs.29.334
PV of Re.1 to be received after 20 years = 0.29334. Consulting the PVF table, we find that the rate of interest in this case is in the range of  6% to 7%.

NPV at  6 % = -29.334 +(100 x 0.312)  = +  1.866
As  NPV (at 6%) is positive, this shows that the return is greater than 6%. Let calculate NPV at 7%.

NPV at  7 % = -29.334 +(100 x 0.258)  = - 3.534
As  NPV ( at 7%) is negative, this shows that the return is less than 7%.
We can find the exact return (called YTM, also called current interest rate) through interpolation.

YTM or current interest rate :
Lower rate NPV

\[
\text{Lower rate NPV} = \text{Lower rate} + \frac{\text{difference in rates}}{\text{Lower rate NPV} - \text{Higher rate NPV}} \\
\]

\[
= 6 + \frac{1.866}{1.866 - (-3.534)} = 6.35\% \\
\]

Q. No. 51: A five year zero coupon bond yields 7%. Find its market price today. After 1 year?

Answer

Suppose the maturity value of the bond = Rs.100

M. price = \( \frac{100}{(1.07)^5} = 71.30 \)

M. price after 1 year = \( \frac{100}{(1.07)^4} = 76.29 \)

Q. No. 52: Determine the price of Rs.1000 zero coupon bond with YTM of 15% and 10 years to maturity.

Answer

Let the price of the bond = X

\[
X (1.15)^{10} = Rs.1,000 \\
X = Rs.1,000 / (1.15)^{10} \\
X = Rs.247 \\
\]

Q. No. 53: Face value Rs.100. Maturity period is 10 years. Coupon rate 13.88% payable semi-annually. Find the market price assuming the yield to maturity is 9% p.a. half yearly compounded.

Answer

Market price = \( (6.94)(13.026) + 100 (0.4165) = 132.05 \)

Q. No. 54: Calculate the durations and volatilities of the debt security A, the cash flows of the security are given below (assume YTM 8%)

<table>
<thead>
<tr>
<th>End of period 1</th>
<th>End of period 2</th>
<th>End of period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Answer

Duration of A

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40 x 0.926</td>
<td>37.04</td>
</tr>
<tr>
<td>2</td>
<td>40 x 0.857</td>
<td>68.56</td>
</tr>
<tr>
<td>3</td>
<td>40 x 0.794</td>
<td>95.28</td>
</tr>
</tbody>
</table>

\[
\text{Duration} = \frac{\sum XW}{\sum W} = 1.95 \\
\text{Volatility} = \% \text{ change in bond price on one percent change in YTM} \\
= - \frac{\text{Duration}/(1+\text{YTM/n})}{\% \text{ change in YTM}} = -\frac{1.95/(1.08)}{1} = -1.81\% \\
\]
[1% change in YMT causes 1.81% change in bond price in opposite direction]

**Q. No. 55**: Madhav buys a bond with 4 years maturity. Face value Rs100. Coupon rate 9%. YTM 9%. What is the duration of the bond? What will be the price of the bond if YTM rises to 10%.

**Answer**

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9 x 0.917</td>
<td>8.253</td>
</tr>
<tr>
<td>2</td>
<td>9 x 0.842</td>
<td>15.156</td>
</tr>
<tr>
<td>3</td>
<td>9 x 0.772</td>
<td>20.844</td>
</tr>
<tr>
<td>4</td>
<td>109 x 0.708</td>
<td>308.688</td>
</tr>
<tr>
<td></td>
<td><strong>ΣW = 99.95</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>ΣXW = 352.941</strong></td>
<td></td>
</tr>
</tbody>
</table>

Duration = \(\frac{ΣXW}{ΣW} = \frac{352.941}{99.95} = 3.53\)

% change in bond price = \(-\frac{3.53}{1.09}\) x (Δ BP/100) = -3.24%

When YTM rises to 10%, the price of the bond decreases by 3.24%. Hence new price = 99.95 – 99.95(0.0324) = Rs.96.71

**Q. No. 56**: The following data are available for a bond:

- Face value: Rs.1,000
- Coupon bonds: 16%
- Years to maturity: 6
- Redemption value: Rs.1,000
- YTM: 17%

What is the current market price, duration and volatility of this bond? Calculate the expected market price, if increase in required yield by 75 basis points. *(Nov. 2005)*

**Answer**

<table>
<thead>
<tr>
<th>X</th>
<th>W</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160 x 0.855 = 136.80</td>
<td>136.80</td>
</tr>
<tr>
<td>2</td>
<td>160 x 0.731 = 116.96</td>
<td>233.92</td>
</tr>
<tr>
<td>3</td>
<td>160 x 0.624 = 99.84</td>
<td>299.52</td>
</tr>
<tr>
<td>4</td>
<td>160 x 0.534 = 85.44</td>
<td>341.76</td>
</tr>
<tr>
<td>5</td>
<td>160 x 0.456 = 72.96</td>
<td>364.80</td>
</tr>
<tr>
<td>6</td>
<td>1,160x0.390 = 452.40</td>
<td>2714.4</td>
</tr>
<tr>
<td>Total</td>
<td>964.40</td>
<td>4091.20</td>
</tr>
</tbody>
</table>

Current market price = Rs.964.40

Duration = 4091.20/964.40 = 4.24

Volatility = % change in bond price = \(-\frac{4.24}{1.17}\) = - 3.62%

% change in bond price = \(-\frac{4.24}{1.17}\) x (Δ BP/100) = - 2.72%
Q. No.57 (a) Consider two bonds, one with 5 years to maturity and the other with 20 years to maturity. Both the bonds have a face value of Rs. 1,000 and coupon rate of 8% (with annual interest payments) and both are selling at par. Assume that the yields of both the bonds fall to 6%, whether the price of bond will increase or decrease? What percentage of this increase/ decrease comes from a change in the present value of bond’s principal amount and what percentage of this increase/decrease comes from a change in the present value of bond’s interest payments? (8 Marks)

(b) Consider a bond selling at its par value of Rs. 1,000, with 6 years to maturity and a 7% coupon rate (with annual interest payment), what is bond’s duration? (6 Marks)

(c) If the YTM of the bond in (b) above increases to 10%, how it affects the bond’s duration? And why? (3 Marks)

(d) Why should the duration of a coupon carrying bond always be less than the time to its maturity? (3 Marks) (June 2009)

Answer

(a) Present yield to maturity = 8%

5 years to maturity:
The present value of bond principal (5 years duration) is \( \frac{1000}{(1.08)^5} \) i.e. Rs.681. Present value of interest payments = 1000–681 = 319

It interest falls to 6%, the bond value will go up.
The present value of bond principal will be \( \frac{1000}{(1.06)^5} \) i.e. 747 i.e. the present value of bond principal will increase by Rs.66.
The present value of interest payments will be: 80 (4.212) = 337 i.e. the present value of interest payments will increase by Rs. 18.

The new value of bond will be 747 + 337 = 1084 i.e. the bond value will increase by Rs.84.

% increase in bond value = 8.40%. There are two components of this %.
(i) % Increase due to present value of principal : \( \frac{66}{84} \times 100 = 78.57 \)
(ii) % Increase due to present value of interest : \( \frac{18}{84} \times 100 = 21.43 \)

Teaching note – not to be given in the exam.
The value of bond increased by 8.40%.
Of this, 78.57% i.e. 8.40 x 78.57/100 i.e. 6.60% increase is due to change in present value of principal amount of bond.
21.43% of this change i.e.8.40 x 21.43/100 i.e. 1.80% increase is due to change in the present value of interest payments.
Total increase = 6.60 + 1.80 i.e. 8.40%

20 years to maturity:
The present value of bond principal (20 years duration) is \( \frac{1000}{(1.08)^{20}} \) i.e. Rs.215. Present value of interest payments = 1000 - 215 = 785

It interest falls to 6%, the bond value will go up. The present value of bond principal will be \( \frac{1000}{(1.06)^{20}} \) i.e. 312 i.e. the present value of bond principal will increase by Rs. 97. The present value of interest payments will be: 80 (11.47) i.e. 918 = i.e. the present value of interest payments will increase by Rs. 133

The new value of bond will be 312 + 918 = 1230 i.e. the bond value will increase by Rs.230

% increase in bond value = 23%. There are two components of this %.

(i) % Increase due to present value of principal : ( 97/230) x 100 = 42.17%
(ii) % Increase due to present value of interest: (133/230) x 100 = 57.83%

(b) Present YTM = 7%

Calculation of Bond duration :

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70 x 0.935</td>
<td>65.45</td>
</tr>
<tr>
<td>2</td>
<td>70 x 0.873</td>
<td>122.22</td>
</tr>
<tr>
<td>3</td>
<td>70 x 0.816</td>
<td>171.36</td>
</tr>
<tr>
<td>4</td>
<td>70 x 0.763</td>
<td>213.64</td>
</tr>
<tr>
<td>5</td>
<td>70 x 0.713</td>
<td>249.55</td>
</tr>
<tr>
<td>6</td>
<td>1070 x 0.666</td>
<td>4275.72</td>
</tr>
<tr>
<td></td>
<td>( \sum W = 999.62^* )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sum XW = 5097.94 )</td>
<td></td>
</tr>
</tbody>
</table>

Duration = \( \frac{\sum XW}{\sum W} = \frac{5097.94}{1000} = 5.098 \)

(c) Calculation of Bond duration :

<table>
<thead>
<tr>
<th>Period (X)</th>
<th>PV of cash-in-flows (W)</th>
<th>XW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70 x 0.909</td>
<td>63.63</td>
</tr>
<tr>
<td>2</td>
<td>70 x 0.826</td>
<td>115.64</td>
</tr>
<tr>
<td>3</td>
<td>70 x 0.751</td>
<td>157.71</td>
</tr>
<tr>
<td>4</td>
<td>70 x 0.683</td>
<td>191.24</td>
</tr>
<tr>
<td>5</td>
<td>70 x 0.621</td>
<td>217.35</td>
</tr>
<tr>
<td>6</td>
<td>1070 x 0.564</td>
<td>3620.88</td>
</tr>
<tr>
<td></td>
<td>( \sum W = 868.78 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sum XW = 4366.45 )</td>
<td></td>
</tr>
</tbody>
</table>

Duration = \( \frac{\sum XW}{\sum W} = \frac{4366.45}{868.78} = 5.026 \)

The duration of the bond decreases as the bond price has reduced while the receipts have remained unchanged.

---

6 This value is due rounding off. Otherwise it should be equal to current price of bond i.e. Rs.1000.
(d) Duration is period during which the amount of investment is recovered. As the amount of receipts is more than the amount of investment (on account of interest), duration is less than maturity period.

**Q. No.58** An investor is considering the purchase of the following bond:

- **Face value**: Rs. 100
- **Coupon rate**: 11%
- **Maturity**: 3 years

(i) If he wants an yield of 13%, what is the maximum price he should be ready to pay for.

(ii) If the bond is selling for Rs.97.60, what should be the yield? (Nov. 2009)

**Answer**

(i) **MP of Bond** = 11(2.361) + 100 x 0.693 = 95.27

(ii) Average return per year per debenture: [(33+2.40) / (3)] = 11.80

Approximate annual rate = [11.80 / 97.60] x100 = 12.09%

NPV at 12 % = -97.60 + (11 x 0.893) + (11 x 0.797 )+ (111 x 0.712)= + 0.02

As the NPV tends to zero, YTM = 12%

**Q. No.59** Sarnam Ltd has issued convertible debentures with coupon rate of 12%. Each debenture has an option to convert 20 equity shares at any time until the maturity. The debentures will be redeemed at Rs.100 after 5 years. An investor generally requires a rate of return of 8% on a 5-years security. As an investor when will you exercise conversion for given market prices of the equity shares of (i) Rs.4 (ii) Rs.5 and (iii) Rs.6.

- Cumulative PV factor for 8% for 5 years : 3.993
- PV factor for 8% for 5 years : 0.681 (Nov. 2009)

**Answer**

Current value of Bond:  12(3.993) + 100 x 0.681 = 116.02

<table>
<thead>
<tr>
<th>Realization on conversion</th>
<th>MP equity share Rs.4</th>
<th>MP equity share Rs.5</th>
<th>MP equity share Rs.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 x 20 = 80</td>
<td>5 x 20 = 100</td>
<td>6 x 20 = 120</td>
</tr>
</tbody>
</table>

Conversion is recommended if the MP of equity share is Rs.6.

**Q. No.60** Based on the credit rating of bonds, Mr. Z has decided to apply the following discount rates for valuing the bonds:

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>364 day T bill rate of 3 % spread</td>
</tr>
<tr>
<td>AA</td>
<td>AAA + 2 % spread</td>
</tr>
<tr>
<td>A</td>
<td>AAA + 3 % spread</td>
</tr>
</tbody>
</table>

He is considering to invest in AA rated Rs.1000 face value bond currently selling at Rs.1025.86. The bond has 5 years to mature and the coupon rate is 15% p.a.
payable annually. The next interest is due one year from today and the bond is redeemable at par. (Assume the 364 day T-bill rate to be 9%)

You are required to calculate the intrinsic value of the bond for Z. should he invest in the bond? Also calculate the current yield and the yield to maturity (YTM) of the bond. (Nov. 2008)

**Answer:**

(i) Intrinsic Value of Bond:

\[ 150 \text{ (annuity for 4 years @ 14\%) + } 1150(\text{PVF5 @14\%}) \]

\[ = 150(2.914) + 1150(0.519) = 1033.95 \]

Mr. Z should invest in the bond as the intrinsic value is more than market value.

(ii) Current yield = \( \frac{150}{1025.86} \times 100 = 14.62\% \)

(iii) Average annual return:

\[ \frac{(750 - 25.86)/5}{1025.86} \times 100 = 14.12\% \]

NPV at 14%

\[ -1025.86 \]

\[ + 150 (2.914) \]

\[ + 1150(0.519) = + 8.09 \]

NPV at 15%

\[ -1025.86 \]

\[ + 150 (2.855) \]

\[ + 1150(0.497) = - 26.06 \]

**YTM:**

\[
\text{Lower rate \ NPV} \\
= \text{Lower rate} + \frac{\text{difference in rates}}{\text{(Lower rate NPV - Higher rate NPV)}} \times \text{(difference in rates)}
\]

\[
= 14 + \frac{8.09}{8.09 - (-26.06)} \times 1 = 14.24\%
\]

**Q. No.61** An investor holds a 10\% Bond maturing after 1 year. The principal amount of the bond is secured through mortgage on the fixed assets of the company and hence, it is fully secured. There is apprehension regarding payment of interest. The probability of default on interest is 0.07. The bond has a face value of Rs.100. The current market price is Rs.98. Find the YTM. What will be the market price if the company’s bankers announce that they
guarantee the payment of interest on these bonds?

**Answer**

\[
\text{YTM} = \frac{109.30}{98} - 1 = 0.1153 = 11.53\%
\]

MP of the bond = \(-\frac{110}{1.1153}\) = Rs.98.63

**Q. No.62** A 5-year maturity 8% bond sells in the market on the basis of 10% YTM. The bond is callable after 3 years at 5% premium over face value. What is YTC?

**Answer**

\[
\text{MP of the bond} = 8(0.909 + 0.826 + 0.751 + 0.683) + 108(0.621) = 92.42
\]

Average return till call = \((24 + 12.58)/3 = 36.58/3 = 12.1933\)  

NPV at 12%
\[
\begin{align*}
&= -92.42 \\
&\quad + 8(0.893 + 0.797 + 0.712) \\
&\quad + 105(0.712) = + 1.556
\end{align*}
\]

NPV at 13%
\[
\begin{align*}
&= -92.42 \\
&\quad + 8(0.885 + 0.783 + 0.693) \\
&\quad + 105(0.693) = -0.767
\end{align*}
\]

\[\text{YTM} : \quad \frac{\text{Lower rate}\cdot\text{NPV}}{\text{Lower rate}\cdot\text{NPV} - \text{Higher rate}\cdot\text{NPV}}\times (\text{difference in rates})\]

\[
\begin{align*}
&= 12 + \frac{1.556}{1.556 - (-0.767)} \\
&= 12.67\%
\end{align*}
\]

**Q.No.63** Fill in the blanks regarding following zero coupon bonds, all have maturity value of Rs.1000.

<table>
<thead>
<tr>
<th>Price</th>
<th>Maturity (Years)</th>
<th>YTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rs.260</td>
<td>8</td>
<td>?</td>
</tr>
<tr>
<td>---</td>
<td>12</td>
<td>15%</td>
</tr>
<tr>
<td>Rs.218</td>
<td>?</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Answer**
(a) Let $YTM = r$
$260(1+r)^8 = 1000$
$(1+r)^8 = 3.8462$
$1+r = 1.1834 \Rightarrow r = 18.34\%$

(b) Let the market price $= x$
$x(1.15)^{12} = 1000 \Rightarrow x = 186.91$

(c) PV of rupee one to be received on maturity (at 10%) = 0.218
Consulting the PVF table, we find that this holds when the maturity is 16 years.

Q. No. 64: The following is the list of prices of zero coupon bonds of various maturities.

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Market price of Rs.1000 face value bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rs.952.38</td>
</tr>
<tr>
<td>2</td>
<td>Rs.890.00</td>
</tr>
<tr>
<td>3</td>
<td>Rs.816.30</td>
</tr>
</tbody>
</table>

What are the forward rates for year 2, and 3?

Answer

Calculation of forward rates using the basic concept of valuation

Year 1: $952.38(1+r) = 1000$
$r = 5\%$

Year 2: $890(1.05)(1+r) = 1,000$
$r = 7.0095\%$

Year 3: $816.30(1.05)(1.070095)(1+r) = 1,000$
$r = 9.03\%$

THEORETICAL ASPECTS

Q.65: What is interest rate risk, reinvestment risk and default risk & what are the types of risk involved in investment in G-sec.? (Nov. 2005)

Answer: The term risk is used to denote the possibility of variability in the returns expected from the investment i.e. the actual return differs from the expected one. Investment in bonds is not entirely risk free. Both systematic and unsystematic risks are associated with the investment in bonds.

Default Risk: The issuer may default in the payment of interest or principal or both on the stipulated dates. This risk is referred as Unsystematic risk of Bond Investments.
Interest Risk: Interest risk refers to change in market interest rate during the holding period. Change in the interest rate causes change in the market price of the Bond. Remember that the bond prices move inversely to market interest rate changes. If interest in the market goes up, the market value of the bond will decline. It is systematic risk of Bond investments.

Reinvestment Risk: Change in market interest rate during the holding period affects the return from the bond investment as the investor shall be reinvesting the interest income of the bond at the changed rate. It is systematic risk of Bond investments.

Two types of risk are involved in G-sec: (i) Interest risk and (ii) reinvestment risk.